

IVA'S CALCULUS I: Homework based on Thursday 3/20 and Friday 3/21 lectures

Purpose

The main purpose of this assignment is to have you practice solving OPTIMIZATION PROBLEMS with calculus techniques. In your textbook this is covered in Section 3.7.

Due date

Monday 3/31/2014, by 5pm.

Problems

1. A two-pen corral is to be built. The outline of the corral forms two identical adjoining rectangles. If there is 120 ft of fencing available, what dimensions of the corral will maximize the enclosed area?
2. An advertisement consists of a rectangular printed region plus 1-inch margins on the sides and 2-in margins at top and bottom. If the area of the printed region is to be 92 square inches, find the dimensions of the printed region and the overall advertisement that minimize the total area.
3. A jewelry box is to be made in the shape of a rectangular box with a square base. The volume of the box needs to be 250 cubic inches. The material used to manufacture the bottom side of the box costs \$1 per square inch. The material for the four sides costs \$2 per square inch and the material for the lid costs \$7 per square inch. Find the dimensions that will minimize the cost of the material to manufacture the box.
4. A cylindrical can is to be made to hold 250 cubic inches. Manufacturing the bottom and the top of the can costs 2 cents per square inch, while manufacturing the side of the can costs 1.5 cents per square inch. Find the dimensions that will minimize the cost of the material to manufacture the can.
5. A box with an open top is to be constructed from a rectangular piece of cardboard 1m wide and 2m long by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.
6. A showroom for a department store is to be rectangular with walls on three sides, 6-ft door opening on the two facing walls and a 10-ft door opening on the remaining wall. The showroom is to have 800 ft² of floor space. What dimensions will minimize the length of wall used?

7. A Norman window has the outline of a semicircle on the top of a rectangle. Suppose there is 12 feet of wood trim available. Find the dimensions of the rectangle (and hence the semicircle) that will maximize the area of the window.
8. A rain gutter with a rectangular cross section is made from a 1-ft \times 20-ft piece of metal by bending up equal amounts from the 1-ft side. How should the metal be bent up on each side in order to make the capacity (i.e volume) of the gutter a maximum?
9. A peach grower has determined that if 30 trees are planted per acre, each tree will average 200 lb of peaches per season. However, for each tree grown in addition to 30 trees the average yield for each of the trees in the grove drops by 5 lb per tree. How many peach trees should be planted on each acre to maximize the yield of peaches per acre? What is the maximum yield?
10. Your favorite branch of your favorite bank is open from 9AM to 6PM. The number of customers the branch had during the first t hours of the work day is given by

$$f(t) = \frac{27}{2}t^2 - t^3.$$

You would like to go do some business in the bank, but you don't want to wait too much. You'd like to avoid the moment during the bank's hours when the rate at which the people are getting into the bank is biggest. Find that moment.